

## The numbers of nature: the Fibonacci sequence

The Fibonacci Sequence has always attracted the attention of people since, as well as having special mathematical properties, other numbers so ubiquitous as those of Fibonacci do not exist anywhere else in mathematics: they appear in geometry, algebra, number theory, in many other fields of mathematics and even in nature! Let's find out together what it is ...

### The life of Fibonacci

Leonardo Pisano, called Fibonacci (Fibonacci stands for *filius Bonacii*) was born in Pisa around 1170. His father, Guglielmo dei Bonacci, a wealthy Pisan merchant and representative of the merchants of the Republic of Pisa in the area of Bugia in Cabilia (in modern north-eastern Algeria), after 1192 took his son with him, because he wanted Leonardo to become a merchant.



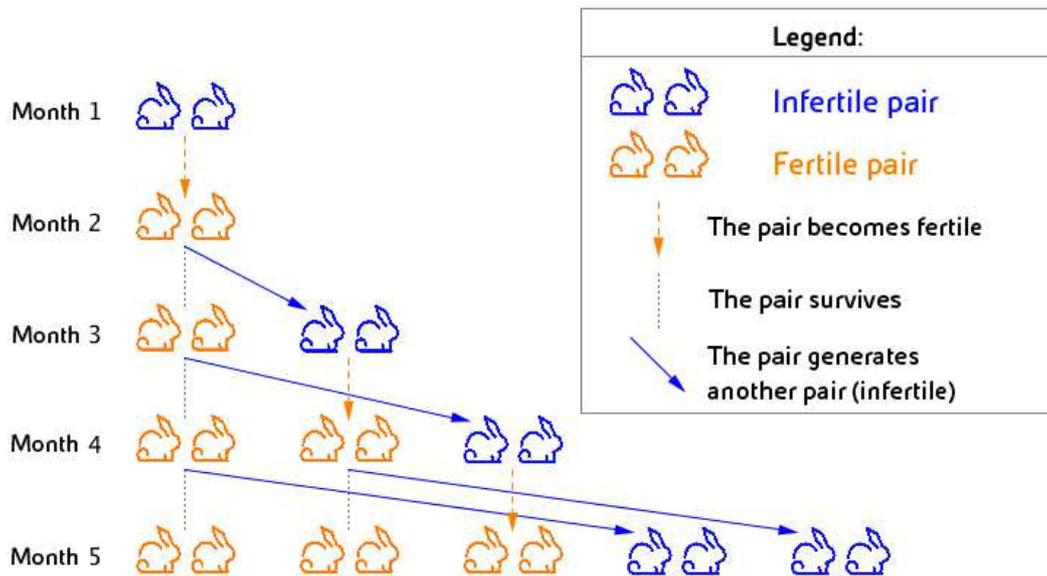
Source: Wikipedia

He thus got Leonardo to study, under the guidance of a Muslim teacher, who guided him in learning calculation techniques, especially those concerning Indo-Arabic numbers, which had not yet been introduced in Europe. Fibonacci's education started in Bejaia and continued also in Egypt, Syria and Greece, places he visited with his father along the trade routes, before returning permanently to Pisa starting from around 1200. For the next 25 years, Fibonacci dedicated himself to writing mathematical manuscripts: of these, *Liber Abaci* (1202), thanks to which Europe became aware of Indo-Arabic numbers, *Practica Geometriae* (1220), *Flos* (1225) and *Liber Quadratorum* (1225) are today known to us. Leonardo's reputation as a mathematician became so great that Emperor Federico II asked an audience while in Pisa in 1225. After 1228, not much is known of Leonardo's life, except that he was awarded the title of "*Discretus et sapiens magister Leonardo Bigollo*" in recognition of the great progress he made to mathematics. Fibonacci died sometime after 1240, presumably in Pisa.

### The rabbits of Fibonacci and the famous sequence

*Liber Abaci*, in addition to referring to Indo-Arabic numbers, which subsequently took the place Roman numerals, also included a large collection of problems addressed to merchants, concerning product prices, calculation of business profit, currency conversion into the various coins in use in the Mediterranean states, as well as other problems of Chinese origin. Alongside these commercial problems were others, much more famous, which also had a great influence on later

authors. Among them, the most famous, source of inspiration for many mathematicians of later centuries, is the following: "How many pairs of rabbits will be born in a year, starting from a single pair, if each month each pair gives birth to a new pair which becomes reproductive from the second month?". The solution to this problem is the famous "Fibonacci sequence": 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89... a sequence of numbers in which each member is the sum of the previous two.



Source: Oilproject

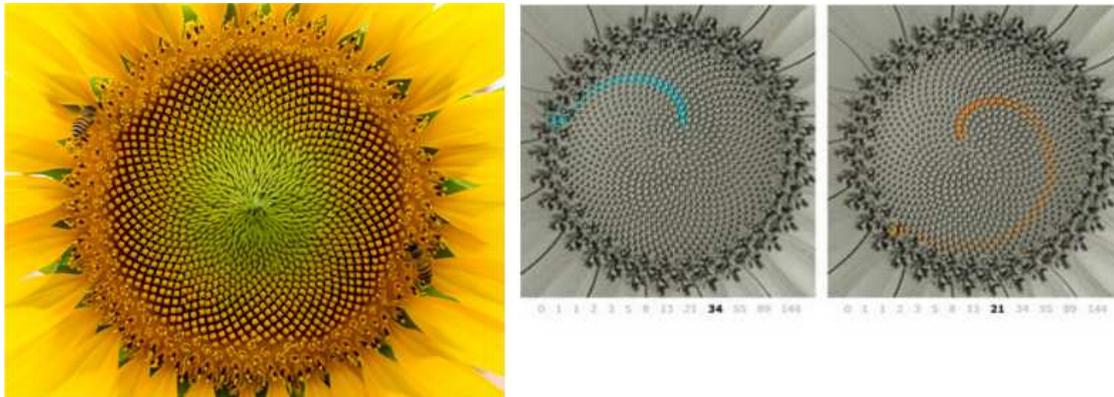
An important characteristic of the sequence is the fact that the ratio between any number and the previous one in the series tends towards a well-defined value: 1.618... This is the golden ratio or golden section,  $\phi$  (Phi), that frequently occurs in nature.

When Fibonacci illustrated this sequence, as a solution to a "recreational mathematics" problem, he did not give it particular importance. Only in 1877 the mathematician Édouard Lucas published a number of important studies on this sequence, which he claimed to have found in *Liber Abaci* and which, in the honour of the author, he called "Fibonacci sequence". Studies subsequently multiplied, and numerous and unexpected properties of this sequence were discovered, so much so that since 1963, a journal exclusively dedicated to it, "The Fibonacci quarterly", has been published.

### The Fibonacci sequence in nature

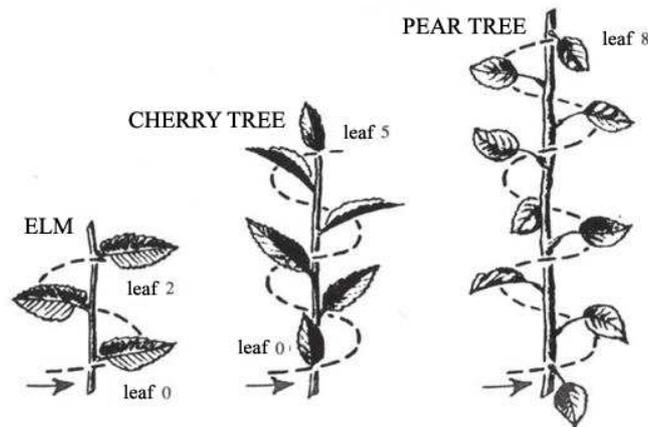
Observing the geometry of plants, flowers or fruit, it is easy to recognize the presence of recurrent structures and forms. The Fibonacci sequence, for example, plays a vital role in phyllotaxis, which studies the arrangement of leaves, branches, flowers or seeds in plants, with the main aim of highlighting the existence of regular patterns. The various arrangements of natural elements follow surprising mathematical regularities: D'arcy Thompson observed that the plant kingdom has a curious preference for particular numbers and for certain spiral geometries, and that these numbers and geometries are closely related.

We can easily find the numbers of the Fibonacci sequence in the spirals formed by individual flowers in the composite inflorescences of daisies, sunflowers, cauliflowers and broccoli.



*In the sunflower, individual flowers are arranged along curved lines which rotate clockwise and counterclockwise. Credits: The Fibonacci sequence in phyllotaxis - Laura Resta (Degree Thesis in biomathematics)*

It was Kepler who noted that on many types of trees the leaves are aligned in a pattern that includes two Fibonacci numbers. Starting from any leaf, after one, two, three or five turns of the spiral there is always a leaf aligned with the first and, depending on the species, this will be the second, the third, the fifth, the eighth or the thirteenth leaf.



*Arrangement of leaves on a stem. Credits: The Fibonacci sequence in phyllotaxis - Laura Resta (Degree Thesis in biomathematics)*

Another simple example in which it is possible to find the Fibonacci sequence in nature is given by the number of petals of flowers. Most have three (like lilies and irises), five (parnassia, rose hips) or eight (cosmea), 13 (some daisies), 21 (chicory), 34, 55 or 89 (asteraceae). These numbers are part of the famous Fibonacci sequence described in the previous paragraph.

*Iris, 3 petals;  
parnassia, 5  
petals; cosmea,  
8 petals*

