Andrew Wiles wins the 2016 Abel Prize for his proof of Fermat's Last Theorem

On 15 March 2016, the President of the Norwegian Academy of Science and Letters, Ole M. Sejersted, at the proposal of the Abel Commission (composed of five internationally renowned mathematicians, currently: John Rognes, Rahul Pandharipande, Éva Tardos, Luigi Ambrosio, Marta Sanz-Solé) has announced the winner of the 2016 Abel Prize: the British mathematician Sir Andrew J. Wiles *"for his stunning proof of Fermat's Last Theorem by way of the modularity conjecture for semistable elliptic curves, opening a new era in number theory".*

The Prize - in memory of the brilliant Norwegian mathematician Niels Henrik Abel - is considered the equivalent of the Nobel Prize and aims to promote mathematics and render it more prestigious. Since 2003, it has been awarded every year to a mathematician who has distinguished himself in the course of his career, and consists of a sum of money of approx. 600,000 euros. In 2015 it was won by John Forbes Nash and Louis Nirenberg (Abel Prize Roll of Honour at the end of the article).

Abel Prize Logo – Credits: www.abelprize.no

The Abel Prize will be awarded at the University of Oslo on 24 May 2016 by Haakon, Crown Prince of Norway (for more details: "*The Abel Prize*").

A compelling theorem

The events related to Fermat's Last Theorem are in some ways compelling; moreover, few mathematical results can boast such a as long and varied history, having now found a solution after more than three and a half centuries. Fermat's Last Theorem refers to the affirmation formulated by Pierre de Fermat in 1637 (who did not make the proof known), written in the margin of a copy of the Arithmetica of Diophantus of Alexandria (250 AD?):

"I have a wonderful proof of this theorem, which cannot be contained in the too narrow margin of the page".

The theorem states that there are no non-zero positive integer solutions to the equation:

$x^n + y^n = z^n$

if n is a number greater than 2 (i.e. $n = 3, 4, 5, \ldots$).

The proof that did not fit in the too narrow margin was never found, and this phrase became the gauntlet picked up by generations of mathematicians, who strove in vain to prove that only apparently so simple theorem.

The Hilbert Problems and the Millennium Problems

Since ancient Greek times, mathematicians have formulated proofs and theorems in the form of solutions to numerical enigmas. Especially in the second half of the nineteenth century, this fashion was also common in the popular press and mathematical problems were found next to crossword puzzles and anagrams. Disseminating the subject with the involvement of the people, mathematicians tried not only to make themselves more "appealing" but also hoped to discover a "misunderstood" mathematical genius, able to provide the solution to an unsolved problem which for years had rattled their brains.
In the wake of this, on 8 August 1900, the German mathematician David Hilbert, during the International Congress of Mathematics held in Paris, presented a list of 23 as yet unsolved mathematical problems (the so-called Hilbert Problems, for more details: "*The Hilbert Problems*").

David Hilbert in 1912 (Credits: Wikipedia).

And exactly a century later, on 24 May 2000, during the Millennium Conference in Paris, the Clay Mathematics Institute brought the Millennium Problems to the attention of mathematicians. Imitating the Hilbert Problems, the institute listed 7

as yet unsolved mathematical problems but, unlike the previous ones, for each of them it established a prize of one million dollars (for more details: "*The Millennium Problems*"). The Riemannè Hypothesis was the only problem in both lists.
Both the Hilbert Problems and the Millennium Problems have had a significant impact on twentieth century mathematics. Today, in the classic formulation of the problems posed by David Hilbert in 1900, problems 3, 7, 10, 11, 13, 14, 17, 18, 19 and 20 have a universally accepted proof, problems 1, 2, 5, 9, 15, 21 and 22 have a solution not accepted by all mathematicians, problems 8 and 12 are unsolved, while problems 4, 6, 16 and 23 are considered to be too vague to reach a solution.

The prince of amateurs

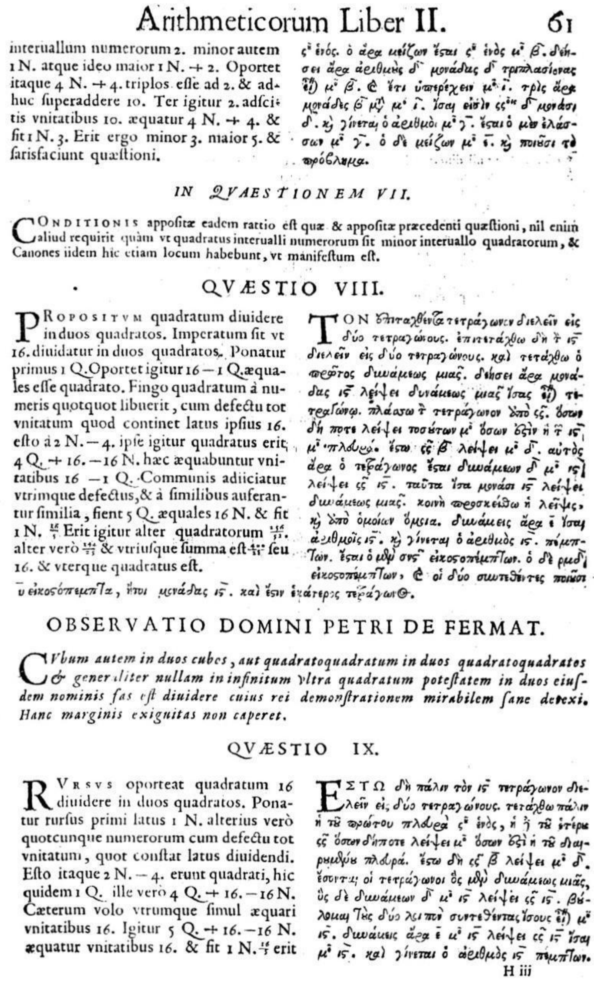
Pierre de Fermat was born on 17 August 1601 in the French town of Beaumont de Lomagne. His father was a wealthy leather merchant and Pierre had the good fortune to enjoy a privileged upbringing, first in a Franciscan monastery and then at the University of Toulouse. Family pressures led him to pursue a career in law but Fermat had no such ambitions and spent his free time and energy on his favourite pastime: mathematics. He was certainly not an academic but a real expert on the subject with a great talent for numbers, earning the nickname "Prince of Amateurs".

Portrait of Pierre de Fermat (Credits: Wikipedia)

While he was engaged in reading Book II of Diophantus' Arithmetica, Fermat came across a number of observations concerning Pythagoras' theorem and the Pythagorean triples. The latter took their name in fact from Pythagoras' theorem: a Pythagorean triple is a set of natural numbers *x, y, x* such that $x^2 + y^2 = z^2$; in fact, every right triangle corresponds to a Pythagorean triple, and vice versa.
Fermat was struck, also because he knew that many centuries before the Greek mathematician *Euclid* (300 BC ?) had developed a proof that showed the existence of an infinite number of Pythagorean triples. While he fantasised on the subject, in a flash of brilliance that was to immortalise the Prince of Amateurs, Fermat created an equation which, although very similar to that of Pythagoras, had no solution. Instead of considering the equation $x^2 + y^2 = z^2$, Fermat considered a variant of Pythagoras' creation: $x^n + y^n = z^n$, where n represents a number greater than 2 (i.e. n = 3, 4, 5, …).
In the margin of his copy of Arithmetica, Fermat jotted down this observation:

"Cubem autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere".

"It is impossible to transcribe a cube as the sum of two cubes or a fourth power as the sum of two fourth powers or, in general, no number that is a power higher than two can be written as the sum of two powers of the same value".



Arithmeticorum Liber II. 61

interuallum numerorum 2. minor autem 1 N. atque ideo maior 1 N. + 2. Oportet itaque 4 N. + 4. triplos eſſe ad 2. & adhuc ſuperaddere 10. Ter igitur 2. adſcitis vnitatibus 10. æquatur 4 N. + 4. & fit 1 N. 3. Erit ergo minor 3. maior 5. & ſatisfaciunt quæſtioni.

IN QVAESTIONEM VII.

CONDITIONIS appoſitæ eadem rattio eſt quæ & appoſitæ præcedenti quæſtioni, nil enim aliud requirit quàm vt quadratus interualli numerorum ſit minor interuallo quadratorum, & Canones iidem hic etiam locum habebunt, vt manifeſtum eſt.

QVÆSTIO VIII.

PROPOSITVM quadratum diuidere in duos quadratos. Imperatum ſit vt 16. diuidatur in duos quadratos. Ponatur primus 1 Q. Oportet igitur 16 – 1 Q. æquales eſſe quadrato. Fingo quadratum à numeris quotquot libuerit, cum defectu tot vnitatum quod continet latus ipſius 16. eſto à 2 N. – 4. ipſe igitur quadratus erit 4 Q. + 16. – 16 N. hæc æquabuntur vnitatibus 16 – 1 Q. Communis adiiciatur vtrimque defectus, & à ſimilibus auferantur ſimilia, fient 5 Q. æquales 16 N. & fit 1 N. $\frac{16}{5}$ Erit igitur alter quadratorum $\frac{256}{25}$. alter verò $\frac{144}{25}$ & vtriuſque ſumma eſt $\frac{400}{25}$ ſeu 16. & vterque quadratus eſt.

OBSERVATIO DOMINI PETRI DE FERMAT.

CVbum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos & generaliter nullam in infinitum vltra quadratum poteſtatem in duos eiuſdem nominis fas eſt diuidere cuius rei demonſtrationem mirabilem ſane detexi. Hanc marginis exiguitas non caperet.

QVÆSTIO IX.

RVRSVS oporteat quadratum 16 diuidere in duos quadratos. Ponatur rurſus primi latus 1 N. alterius verò quotcunque numerorum cum defectu tot vnitatum, quot conſtat latus diuidendi. Eſto itaque 2 N. – 4. erunt quadrati, hic quidem 1 Q. ille verò 4 Q. + 16. – 16 N. Cæterum volo vtrumque ſimul æquari vnitatibus 16. Igitur 5 Q. + 16. – 16 N. æquatur vnitatibus 16. & fit 1 N. $\frac{16}{5}$ erit

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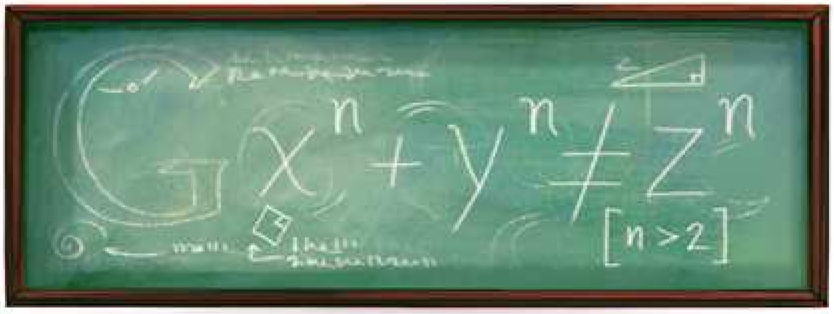
Diophantus' Arithmetica with the Observations of Pierre de Fermat. Credits: [Wikipedia]

Fermat triples

Of all the possible numbers there seemed no reason why one could not find at least one solution, but Fermat affirmed that never in the infinite universe of numbers was there a Fermat triple. It was an extraordinary and assertive affirmation, but one which Fermat thought he could prove, as indicated in the comment that was to become the obsession of generations of mathematicians:

"Cuius rei demonstrationem mirabilem sane detexi hanc marginis exiguitas non caperet".

"I have a wonderful proof of this theorem, which cannot be contained in the too narrow margin of the page".



Google doodle of 17 August 2011 celebrating 410 years since the birth of the mathematician Pierre de Fermat – *Credits:* [*www.google.com/doodles/*](http://www.google.com/doodles/)

From his words one grasps his great satisfaction with his proof but, at the same time, lack of intention to write it down and publish it. Fermat never spoke to anyone about his proof, but Fermat's Last Theorem, as it became known, was destined to become famous worldwide in the following centuries.

If it were not for the commitment and efforts of his eldest son, Clement-Samuel, who had always appreciated the importance of his father's hobby, the discoveries of Fermat, following his death (9 January 1665), would have been lost forever due to his isolation from the spheres of mathematicians. Clement-Samuel Fermat took five years to gather the notes and letters of his father and to examine the annotations in the margins of a copy of Arithmetica, to then publish them in a special edition. In 1670 in Toulouse, Diophantus' Arithmetica with the Observations of P. de Fermat was published and thus Fermat's Observations became known yo the wider scientific community. Together with the Greek original and the Latin translation of Bachet, forty-eight Observations of Fermat appeared, the second destined to become known as Fermat's Last Theorem (from "Fermat's Last Theorem", 1997 by Simon Singh).

The little mathematician

Over the centuries many mathematicians dedicated themselves to the pursuit of proving the theorem and there were also many failures. Among the most famous results we recall Euler, who formulated a proof valid only for n=3, Adrien-Marie Legendre, who solved the case of *n=5* and Sophie Germain, who discovered that it was probably true for *n* equal to a particular prime number *p*, such that *2p + 1* is also prime (the so-called Germain primes).

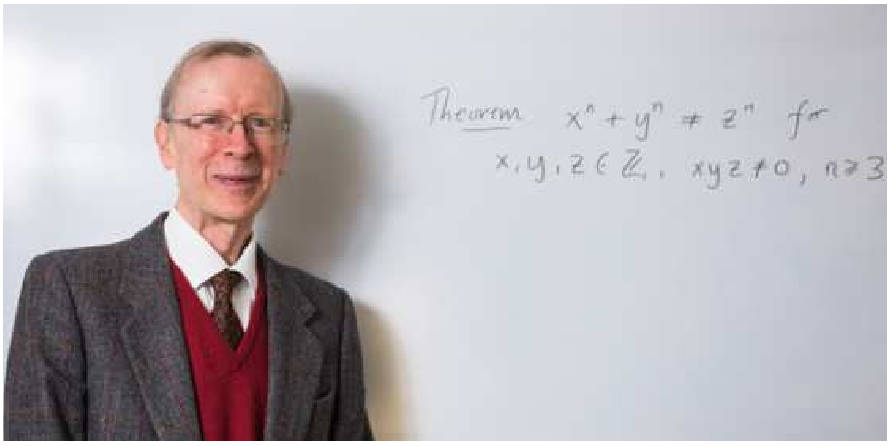
In 1963, the young Andrew Wiles, just ten years of age, was in the library of his Primary School Milton Road (Cambridge) reading a rather special book for a child of that age: The last problem, by Eric Temple Bell, which told the story of Fermat's Last Theorem. The little mathematician was struck by the equation $x^n + y^n = z^n$ and by the affirmation of the Prince of Amateurs nearly three centuries earlier.

To quote his words:

"I found this problem that had remained unsolved for three hundred years. It did not seem that my classmates had a crush on mathematics and so I did not speak with them. But I had a teacher who had done research in mathematics and who gave me a book on number theory; that text provided me with some guidance on how to begin to address the problem. To begin with, I worked on the idea that Fermat did not know much more mathematics than I did". (from *"Fermat's Last Theorem", 1997 by Simon Singh*)

Since then, Wiles never abandoned that theorem and after his studies in mathematics at Oxford and Cambridge, in 1982 he became a professor at Princeton in the United States. In 1985 he decided to entirely dedicate himself to the search for

the proof of Fermat's Last Theorem. This work kept him isolated until 1992, when he thought he was close to completion of the proof. In June 1993, in fact, he announced three seminars at the Newton Institute at Cambridge University, the last of which, on 23 June, took place in a lecture theatre full of enthusiastic mathematicians. The first version of the proof, however, contained a number of serious shortcomings that forced Wiles to return to work to validate all the deductive connections.
With the contribution of his first student, Richard Taylor, Wiles overcame the difficulties, proving the theorem after more than a year, on 19 September 1994. Only in 1998 was Wiles' proof of Fermat's Last Theorem officially accepted by the International Mathematical Union.



Sir Andrew Wiles (Photo: John Cair – Credits: www.abelprize.no)

Wiles was little more than 40 years old when he achieved his result and could not win the Fields Medal, a prize awarded every four years to mathematicians under the age of 40, during the International Congress of Mathematicians. But over the years the British mathematician has certainly not lacked recognitions of all kinds: Schock Prize and Prix Fermat in 1995, British Royal Medal, Cole Prize of the A.M.S. and Wolf Prize in 1996, Special Prize of the International Mathematical Union (IMU) in 1998.
After over 20 years since his discovery, Sir Andrew Wiles has been awarded the 2016 Abel Prize. His proof combines three highly complex fields of mathematics, using algebraic geometry tools, the Galois theory, the theory of elliptic curves and modular forms, a result that many consider more important than Fermat's Last Theorem itself.
Currently Wiles is continuing to work on other unsolved mathematical problems and is apparently dedicating himself to the Birch and Swinnerton-Dyer Conjecture, one of the seven Millennium Problems, according to the Clay Mathematics Institute in Oxford (for more details: "*The Millennium Problems*").

Abel Prize Roll of Honour

2003: **Jean-Pierre Serre**, *"for having played a key role in giving a modern form to numerous branches of mathematics, including topology, algebraic geometry and number theory".*

2004: **Michael F. Atiyah and Isadore M. Singer**, *"for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and for the extraordinary role they played in building new bridges between mathematics and theoretical physics".*

2005: **Peter D. Lax**, *"for his extraordinary contributions to the theory and application of partial differential equations and the computation of their solutions".*

2006: **Lennart Carleson**, *“for his extensive and innovative contributions to harmonic analysis and smooth dynamical systems”.*
2007: **S. R. Srinivasa Varadhan**, *“for his fundamental contributions to probability theory and in particular for creating a unified theory of large deviations”.*
2008: **John Griggs Thompson and Jacques Tits**, *“for their extraordinary achievements in algebra and in particular for their contribution to modern group theory”.*
2009: **Mikhail Gromov**, *“for his revolutionary contributions to geometry”.*
2010: **John Tate**, *“for his work of extensive and lasting impact on number theory”.*
2011: **John Milnor**, *“for his pioneering discoveries in topology, geometry and algebra”.*
2012: **Endre Szemerédi**, *“for his fundamental contributions to discrete mathematics and theoretical computer science, and their profound and lasting impact on additive number theory and ergodic theory”.*
2013: **Pierre Deligne**, *“for the powerful concepts, ideas, methods and results that continue to influence the development of algebraic geometry, as well as all of mathematics”.*
2014: **Yakov G. Sinai**, *“for his fundamental contributions to dynamic systems, ergodic theory and mathematical physics”.*
2015: **John Forbes Nash and Louis Nirenberg**, *“for the significant contributions to differential equation theory, non-linear partial derivatives and their applications to geometric analysis”.*

by Enzo Scasciamacchia

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